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Title: Linearized three-phase optimal power flow models for distribution grids

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Linearized Three-Phase Optimal Power Flow Models for Distribution Grids

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Mentors: David Fobes & Russell Bent

Student Lightning Talks

10 August 2021

About me

- **Current education**: Fourth year PhD student at University of Wisconsin-Madison
- **Research interests**: Optimization and control of power systems
- **Hobbies**: Hiking, video games, badminton and chess

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Outline

Theoretical Background

Work Done

Next Steps

Outline

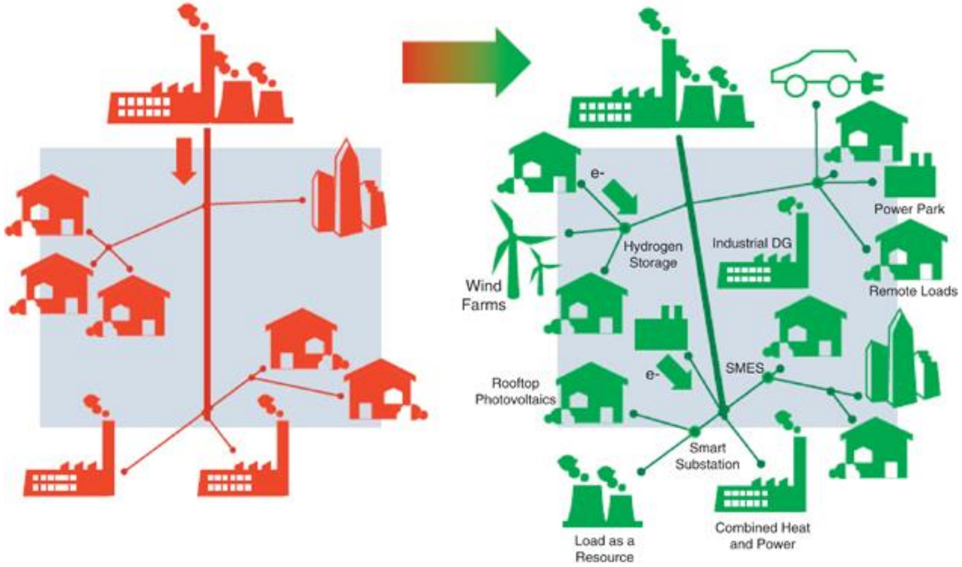
Theoretical Background
- Distribution Grids
- Optimal Power Flow

Work Done

Next Steps

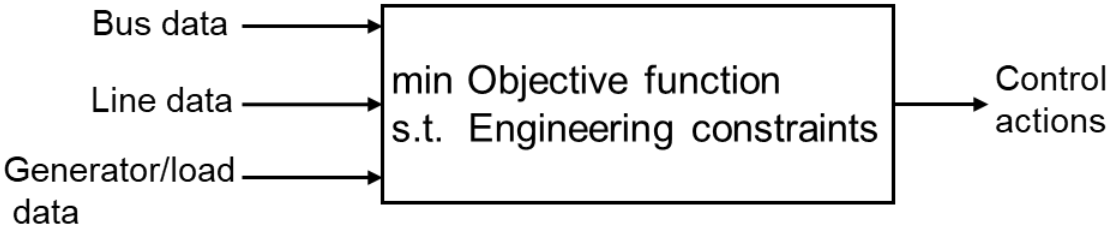
Distribution Grids

- Interconnection between transmission grid and end consumers
- Characteristics different from transmission grid
  - Distributed energy resources (DERs)
  - Radial or near radial structure
  - Unbalanced three-phase systems
    - Untransposed lines with high R/X ratio
    - Asymmetrical loads



Optimal Power Flow (OPF)

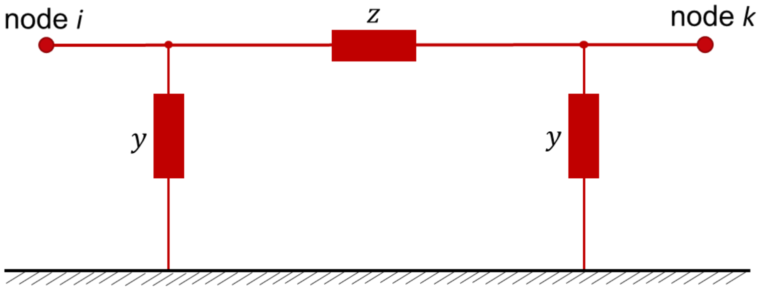
- Optimization problem solving power flow while also optimizing operating conditions and adjusting control actions



- PowerModelsDistribution.jl- Julia package for steady-state distribution grid optimization
  - PowerModels.jl- steady-state transmission grid optimization

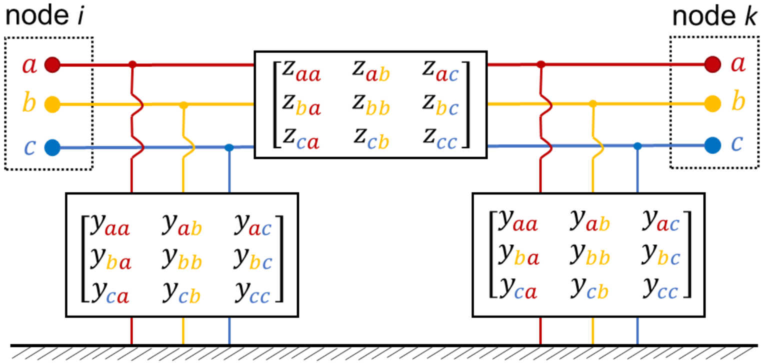
PowerModels vs PowerModelsDistribution

Transmission line



- Balanced system

Distribution line



- Unbalanced system with interphase coupling

Three-phase Optimal Power Flow

- Nonlinear, nonconvex optimization problem solved using Ipopt

min Cost of generation, network losses → Objective function

s.t. DER limits,
Power flow limits,
Voltage magnitude limits,
Nonlinear three-phase AC power balance → Engineering constraints

- Problem: Challenging to solve for large, realistic distribution grids ☹
- Solution: Reduce computational complexity using approximations/relaxations ☺

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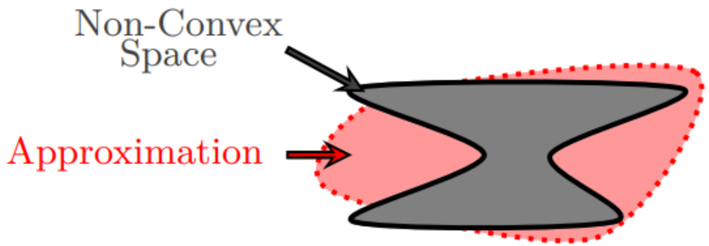
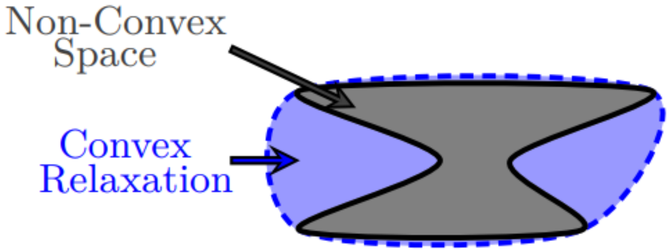
Approximations vs Relaxations





Molzahn, Daniel K., and Ian A. Hiskens. "A survey of relaxations and approximations of the power flow equations." *Foundations and Trends® in Electric Energy Systems* 4.1-2 (2019): 1-221.

- Does not enclose non-convex feasible space
- More computationally tractable
- Example: LinDistFlow

- Enclose non-convex feasible space
- Inefficient scaling for large systems
- Restrictions on possible objectives
- Example: SDP, SOC

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- Linear Approximations

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Nonlinear AC Power Balance Equations

- Satisfy power balance at all nodes (polar coordinate frame)

Network parameters

Voltage variables

$$\boldsymbol{P}_i = |\boldsymbol{V}_i| \odot \sum_{k \in \mathcal{N}} \left[G_{ik} \odot C\big(\boldsymbol{\theta}_{ik}\big) + B_{ik} \odot S\big(\boldsymbol{\theta}_{ik}\big) \right] \cdot |\boldsymbol{V}_k|$$

$$\boldsymbol{Q}_i = |\boldsymbol{V}_i| \odot \sum_{k \in \mathcal{N}} \left[G_{ik} \odot S\big(\boldsymbol{\theta}_{ik}\big) - B_{ik} \odot C\big(\boldsymbol{\theta}_{ik}\big) \right] \cdot |\boldsymbol{V}_k|$$

$$|\boldsymbol{V}_i| = \begin{bmatrix} |\boldsymbol{V}_i^a| \\ |\boldsymbol{V}_i^b| \\ |\boldsymbol{V}_i^c| \end{bmatrix} \quad \boldsymbol{\theta}_i = \begin{bmatrix} \boldsymbol{\theta}_i^a \\ \boldsymbol{\theta}_i^b \\ \boldsymbol{\theta}_i^c \end{bmatrix}$$

Generation-Load

$$C\big(\boldsymbol{\theta}_{ik}\big) = \begin{bmatrix} \cos(\boldsymbol{\theta}_i^a - \boldsymbol{\theta}_k^a) & \cos(\boldsymbol{\theta}_i^a - \boldsymbol{\theta}_k^b) & \cos(\boldsymbol{\theta}_i^a - \boldsymbol{\theta}_k^c) \\ \cos(\boldsymbol{\theta}_i^b - \boldsymbol{\theta}_k^a) & \cos(\boldsymbol{\theta}_i^b - \boldsymbol{\theta}_k^b) & \cos(\boldsymbol{\theta}_i^b - \boldsymbol{\theta}_k^c) \\ \cos(\boldsymbol{\theta}_i^c - \boldsymbol{\theta}_k^a) & \cos(\boldsymbol{\theta}_i^c - \boldsymbol{\theta}_k^b) & \cos(\boldsymbol{\theta}_i^c - \boldsymbol{\theta}_k^c) \end{bmatrix} \qquad S\big(\boldsymbol{\theta}_{ik}\big) = \begin{bmatrix} \sin(\boldsymbol{\theta}_i^a - \boldsymbol{\theta}_k^a) & \sin(\boldsymbol{\theta}_i^a - \boldsymbol{\theta}_k^b) & \sin(\boldsymbol{\theta}_i^a - \boldsymbol{\theta}_k^c) \\ \sin(\boldsymbol{\theta}_i^b - \boldsymbol{\theta}_k^a) & \sin(\boldsymbol{\theta}_i^b - \boldsymbol{\theta}_k^b) & \sin(\boldsymbol{\theta}_i^b - \boldsymbol{\theta}_k^c) \\ \sin(\boldsymbol{\theta}_i^c - \boldsymbol{\theta}_k^a) & \sin(\boldsymbol{\theta}_i^c - \boldsymbol{\theta}_k^b) & \sin(\boldsymbol{\theta}_i^c - \boldsymbol{\theta}_k^c) \end{bmatrix}$$

First-order Taylor (FOT) Approximation

- Linearized power balance equations

Jacobian

$$\begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \end{bmatrix} = \boxed{\begin{bmatrix} P_0 \\ Q_0 \end{bmatrix}} + \boxed{J_0} \cdot \begin{bmatrix} |\boldsymbol{V}| - \boxed{V_0} \\ \boldsymbol{\theta} - \boxed{\theta_0} \end{bmatrix}$$

Initial operating point

$$J = \begin{bmatrix} \frac{\delta \boldsymbol{P}}{\delta |\boldsymbol{V}|} & \frac{\delta \boldsymbol{P}}{\delta \boldsymbol{\theta}} \\ \frac{\delta \boldsymbol{Q}}{\delta |\boldsymbol{V}|} & \frac{\delta \boldsymbol{Q}}{\delta \boldsymbol{\theta}} \end{bmatrix}$$

Forward-Backward Sweep (FBS)

- Backward Sweep (KCL)

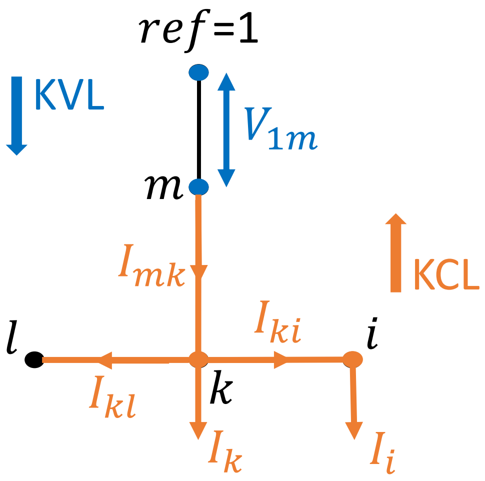
$$\boldsymbol{I}_i = \boldsymbol{I}_{ki} = \frac{(\boldsymbol{P}_i - j\boldsymbol{Q}_i)}{\boxed{\overline{V}_{i0}}}$$

Initial operating point

$$\boldsymbol{I}_{mk} = \boldsymbol{I}_k + \sum_{n \in \{i,l\}} \boldsymbol{I}_{kn}$$

- Forward Sweep (KVL)

$$\boldsymbol{V}_m = \boldsymbol{V}_1 - \underbrace{Z_{1m} \cdot \boldsymbol{I}_{1m}}_{V_{1m}}$$



Summary

Method	Advantages	Limitations
Three-phase OPF	Converges to AC feasible solution	Large computation time
FOT-OPF	Best local approximator	Jacobian calculation can be time-consuming
FBS-OPF	Faster than FOT since it exploits radial topology	Less accurate compared to FOT

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Next Steps

- Testing performance of linear approximations on large, realistic distribution grids
  - Compare solution accuracy and computation time
  - Use different initial operating points
- Warm-starting three-phase OPF to check for improvement in computation time

Thank you for your attention!

With great **power** comes great **responsibility** …

… and greater **electricity bill** !!

Questions